CIE 444 – SOIL MECHANICS

EXAM No. 2

Date: January 24, 2008

Before you start solving the problems, please read the problem questions carefully and take note of the following:

1- You have two hours to solve this exam.

2- The exam is closed book. **A list of useful equations is provided to you with each problem.**

3- Make sure you understand the problem question clearly before you start solving.

4- Be concise, clear and logical in your answers and computations. Justify any assumptions if needed. Points will be deducted for answers that are not supported by proper calculations.

5- Make sure to answer all questions on this booklet.

GOOD LUCK!

NAME:________________________ **ID#:**______________________

Q1:	(20pts)
Q2:	(30pts)
Q3:	(25pts)
Q4:	(25pts)
Total:	**(100pts)**

FINAL GRADE:__________________________

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QUESTION 1: (20pts)

Answer by True (T) or False (F) and correct the statement where needed.

( ) Capillary rise is greater in sands than clays.
F (Capillary rise is greater in clays ~few meters versus few centimetres.)

( ) The coefficient of lateral earth pressure is very sensitive to the geologic and engineering history of a soil.
T

( ) The principle planes are the planes where the shear stress has the maximum value.
F (shear stress is zero)

( ) The graphical representation for any stress state $(\sigma_{11}, \sigma_{33}, \sigma_{13})$ is known as the Mohr Circle.
T

( ) Any deformation of the soil skeleton happens only in response to total stress changes.
F (effective stress changes)

( ) One way to represent the shear strength of a soil is using the Mohr-Coulomb failure criterion.
T

( ) There is no lateral confinement in the triaxial test.
F (direct shear test)

( ) In the consolidation test, the specimen is compressed while maintaining its cross-section constant.
T

( ) A direct shear test clearly shows that dense sands exhibit dilative behavior which is explained by particle interlocking.
T

( ) The critical state friction angle of a dense specimen is the highest value of friction angle a soil can have.
F (peak friction angle)

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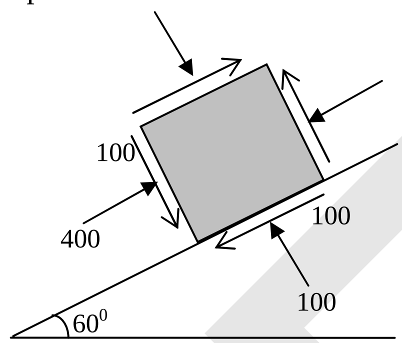
QUESTION 2: (30pts)

Consider the state of stress at a point in a soil mass shown in the figure.



Answer the following using the equations provided below:

$$\sigma_\theta = \frac{1}{2}(\sigma_{11}+\sigma_{33}) + \frac{1}{2}(\sigma_{11}-\sigma_{33})\cos 2\theta + \sigma_{13}\sin 2\theta$$

$$\tau_\theta = \frac{1}{2}(\sigma_{11}-\sigma_{33})\sin 2\theta - \sigma_{13}\cos 2\theta$$

a) What are the values of σ_{11}, σ_{33} and σ_{13} to use in the equations of σ_θ and τ_θ? (3pts)

$$\sigma_{11} = 400kPa$$

$$\sigma_{33} = 100kPa$$

$$\sigma_{13} = -100kPa$$

b) What are the values of σ_θ and τ_θ acting on the horizontal plane? (7pts)

Horizontal plane $\Rightarrow \theta = 30^o$ C.C.W. from π_{11} plane

$$\sigma_\theta = \frac{1}{2}(\sigma_{11}+\sigma_{33}) + \frac{1}{2}(\sigma_{11}-\sigma_{33})\cos 2\theta + \sigma_{13}\sin 2\theta$$

$$\Rightarrow \sigma_\theta = \frac{1}{2}(400+100) + \frac{1}{2}(400-100)\cos 60^o - 100\sin 60^o = 238.4kPa$$

$$\tau_\theta = \frac{1}{2}(\sigma_{11}-\sigma_{33})\sin 2\theta - \sigma_{13}\cos 2\theta$$

$$\Rightarrow \tau_\theta = \frac{1}{2}(400-100)\sin 60^o + 100\cos 60^o \approx 180kPa$$

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c) What are the directions of the major and minor principal planes? (6pts)

To find the direction of the principal planes, the shear stress should be zero.

$$\tau_{\theta_p} = 0 \Rightarrow \tan\left(2\theta_p\right) = \frac{2\sigma_{13}}{\sigma_{11} - \sigma_{33}} = \frac{2(-100)}{(400-100)}$$

$$2\theta_p = -33.69^o$$

$$\theta_{p1} = \frac{-33.69^o}{2} = -16.84^o$$

and $\theta_{p3} = -16.84^o + 90^o = 73.16^o$

d) What are the principal stresses? (6pts)

$$\sigma_1 = \frac{1}{2}(\sigma_{11} + \sigma_{33}) + \frac{1}{2}(\sigma_{11} - \sigma_{33})\cos 2\theta_{p1} + \sigma_{13}\sin 2\theta_{p1}$$

$$\Rightarrow \sigma_1 = \frac{1}{2}(400+100) + \frac{1}{2}(400-100)\cos\left(-33.69^o\right) - 100\sin\left(-33.69^o\right) \approx 430kPa$$

$$\sigma_3 = \frac{1}{2}(\sigma_{11} + \sigma_{33}) + \frac{1}{2}(\sigma_{11} - \sigma_{33})\cos 2\theta_{p3} + \sigma_{13}\sin 2\theta_{p3}$$

$$\Rightarrow \sigma_1 = \frac{1}{2}(400+100) + \frac{1}{2}(400-100)\cos\left(2\times 73.16^o\right) - 100\sin\left(2\times 73.16^o\right) \approx 70kPa$$

e) Solve the entire problem using a Mohr circle and the pole method. (8pts)

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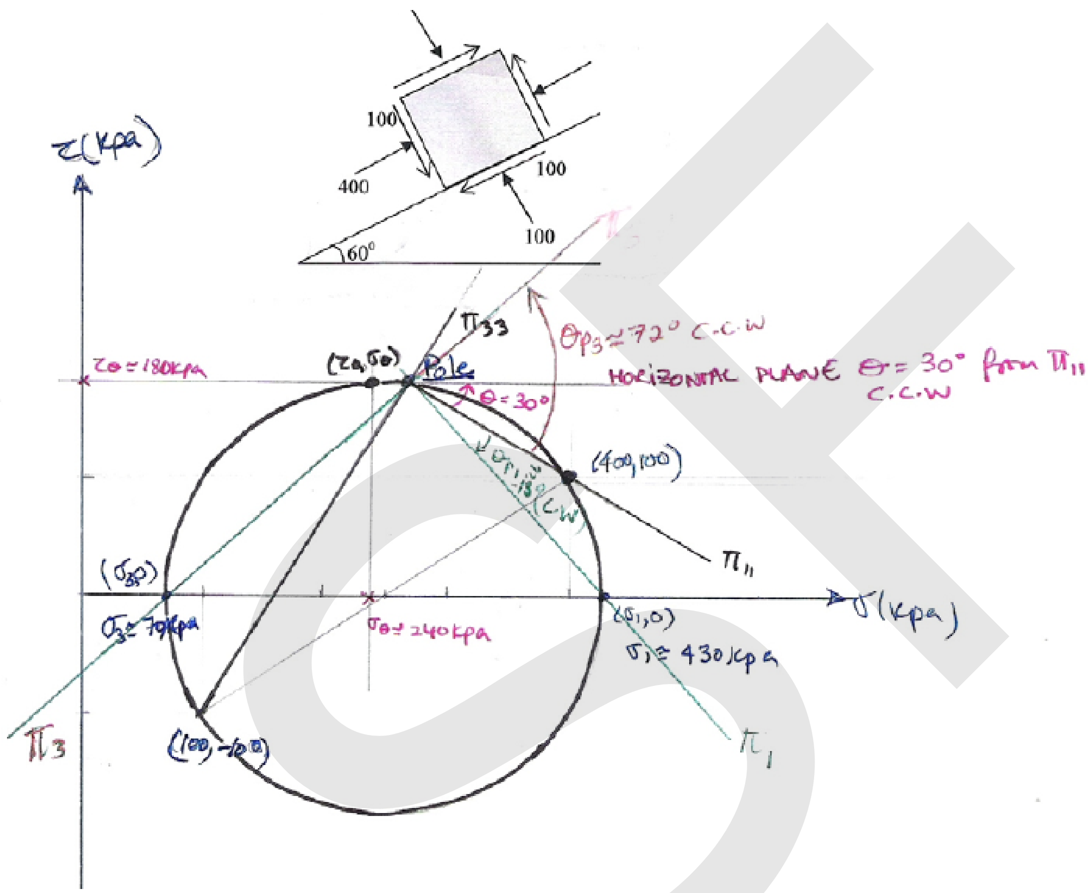

100
400
100
100
60°
τ(kpa)
σ(kpa)
Pole
HORIZONTAL PLANE θ = 30° from π11 C.C.W
θ = 30°
(400,100)
(100,-100)
σ3 ≈ 70kpa
σθ ≈ 240kpa
σ1 ≈ 430kpa
τθ ≈ 180kpa
π11
π33
π1
π3

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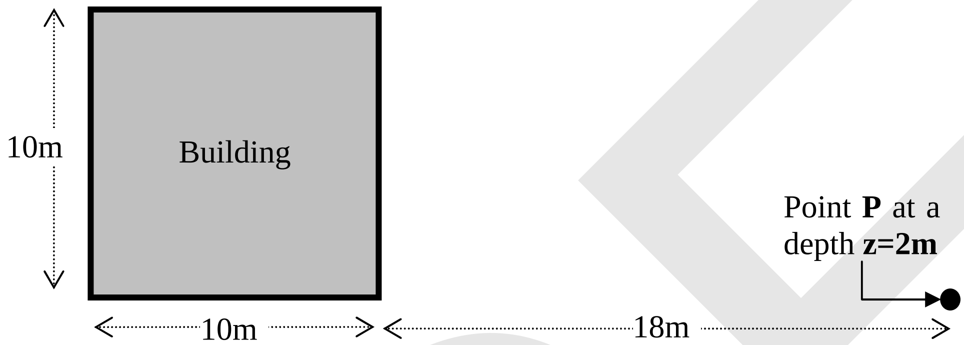
QUESTION 3: (25 pts)

A small building weighs Q=30,000 kN and has a square 10x10m shape. The distributed uniform load under this building is thus equal to q_b=300 kPa. Calculate the vertical stress at point P as shown in the figure:



(a) using the equation for rectangular uniform loads (10pts)

$$\sigma_{z(corner)} = \frac{q_b}{4\pi}\left[\frac{2mn\sqrt{C_1}}{C_1+C_2}\left(\frac{1+C_1}{C_1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{C_1}}{C_1-C_2}\right)\right]$$

where

$$m=\frac{B}{z}$$

$$n=\frac{L}{z}$$

$$C_1=m^2+n^2+1$$

$$C_2=(mn)^2$$

<u>Note:</u> $\tan^{-1}\left(\frac{2mn\sqrt{C_1}}{C_1-C_2}\right)$ **must give a positive angle <u>in radians</u>, otherwise you must add π to that angle**

(b) using the Boussinesq equation (10pts)

$$\sigma_z=\frac{3Qz^3}{2\pi\left(r^2+z^2\right)^{5/2}}$$

where ***r*** is the radial distance from the point of application of the load to point P.

(c) Discuss your results (5pts)

(a) using the equation for rectangular uniform loads

$$\sigma_p = \sigma_{(18\times10)} - \sigma_{(18\times10)} \text{ at the corner}$$

Area (1): 28x10

$$m = \frac{B}{z} = \frac{10}{2} = 5$$

$$n = \frac{L}{z} = \frac{28}{2} = 14$$

$$C_1 = m^2 + n^2 + 1 = 222$$

$$C_2 = (mn)^2 = 4900$$

$C_2 > C_1 \Rightarrow$ need to add π to the $\tan^{-1}\left(\frac{2mn\sqrt{C_1}}{C_1 - C_2}\right)$ term

$$\sigma_{(28\times10)} = \frac{300}{4\pi}\left[\frac{2\times5\times14\sqrt{222}}{222+4900}\left(\frac{1+222}{222}\right) + \tan^{-1}\left(\frac{2\times5\times14\sqrt{222}}{222-4900}\right) + \pi\right] = 74.75 kN/m^2$$

Area (1): 18x10

$$m = \frac{B}{z} = \frac{10}{2} = 5$$

$$n = \frac{L}{z} = \frac{18}{2} = 9$$

$$C_1 = m^2 + n^2 + 1 = 107$$

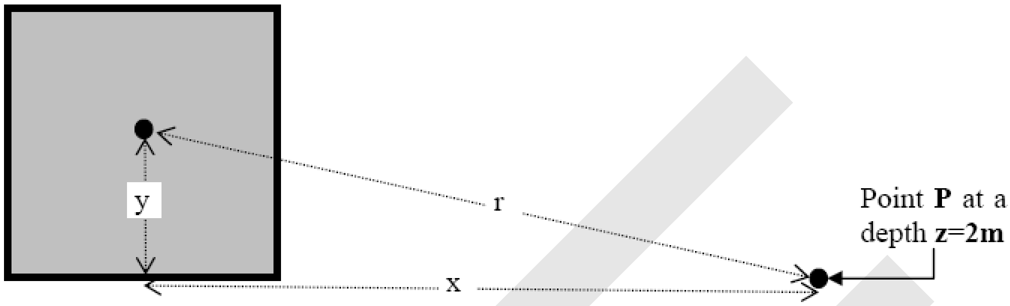
$$C_2 = (mn)^2 = 2025$$

$C_2 > C_1 \Rightarrow$ need to add π to the $\tan^{-1}\left(\frac{2mn\sqrt{C_1}}{C_1 - C_2}\right)$ term

$$\sigma_{(18\times10)} = \frac{300}{4\pi}\left[\frac{2\times5\times9\sqrt{107}}{107+2025}\left(\frac{1+222}{222}\right) + \tan^{-1}\left(\frac{2\times5\times9\sqrt{107}}{107-2025}\right) + \pi\right] = 74.73 kN/m^2$$

$$\boxed{\sigma_p = \sigma_{(28\times10)} - \sigma_{(28\times10)} = 74.75 - 74.73 = 0.02 kN/m^2}$$

(b) using the Boussinesq equation



$r^2 = x^2 + y^2 = 554m$

$$\sigma_p = \frac{3Qz^3}{2\pi\left(r^2 + z^2\right)^{5/2}} = \frac{3 \times 30000 \times 2^3}{2\pi\left(554 + 4\right)^{5/2}}$$

$$\boxed{\sigma_p = 0.016 kN/m^2}$$

(c) Discuss your results

Comparing the results, we can see that both methods give approximately similar results. We can thus say that Boussinesq's solution can be used to approximate the uniform load solution when the point is far from the applied load, i.e., does not feel the load.

This is known as **St. Venant's Principle.**

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QUESTION 4: (25pts)

(a) A direct shear test was performed on a sample of dry clean sand at an initial relative density of D_R = 80%. Conditions at peak strength are 100kPa of vertical stress and 90kPa of shear stress applied in the horizontal direction. What is the peak friction angle based on this test? (5pts)

At failure, $\tau = \sigma \tan\phi$

Given, $\tau_{peak} = 90kPa$, $\sigma_{peak} = 100kPa \Rightarrow \tan\phi_{peak} = \dfrac{90}{100}$

$$\Rightarrow \phi_{peak} = 42^o$$

(b) A fully drained triaxial test is performed on a sample of dry clean sand prepared at the same initial relative density of D_R = 80%. At peak strength, the major principal (vertical) stress is 287kPa and the minor principal (lateral) stress is 67kPa. What is the peak friction angle based on this test? (5pts)

At failure, $\dfrac{\sigma_1}{\sigma_3} = N = \dfrac{1+\sin\phi}{1-\sin\phi}$

Given $\sigma_{1,peak} = 287kPa$, $\sigma_{3,peak} = 67kPa$

$$\Rightarrow \frac{\sigma_{1,peak}}{\sigma_{3,peak}} = \frac{1+\sin\phi_{peak}}{1-\sin\phi_{peak}} = \tan^2\left(45 + \frac{\phi_{peak}}{2}\right)$$

$$\Rightarrow \phi_{peak} = 38.4^o$$

(c) Compare the values of peak friction angles obtained from parts (a) and (b). Are these values reasonable? (5pts)

From (a) and (b) we see that $\phi_{peak,DS} > \phi_{peak,TX}$. The result reasonably shows that the direct shear test gives higher friction angle because of two reasons:

1- Lack of lateral confinement
2- Failure plane is not necessarily the weakest plane.

(d) Is the value of peak friction angle obtained from the triaxial test consistent with what Bolton's equation would predict for a ϕ_c of 33 degrees? (10pts)

Using Bolton's Equation, $\phi_p = \phi_c + A_\psi I_R$ **with** $\phi_c = 33^o$ **and** $A_\psi = 3$

$$\sigma'_{mp} = \frac{1}{3}\left(\sigma'_{1peak} + 2\sigma'_{3peak}\right) = \frac{1}{3}\left(287 + 2\times 67\right) = 140.33kPa$$

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$$\Rightarrow I_R = \frac{D_R}{100}\left[Q - \ln\left(\frac{100\sigma'_{mp}}{p_A}\right)\right] - R = \frac{80}{100}\left[10 - \ln\left(\frac{100\times 140.33}{100}\right)\right] - 1 = 3.045$$

$$\Rightarrow \phi_p = \phi_c + A_\psi I_R = 33 + 3\times 3.045 = 42.13^o$$

Bolton's equation predicts a higher peak friction angle than that obtained from the triaxial test. This may be an indication that the critical state friction angle is less than the assumed 33^o.

Given Equations:

$$\frac{\sigma_1}{\sigma_3} = N = \frac{1+\sin\phi}{1-\sin\phi}$$

$$\phi_p = \phi_c + A_\psi I_R$$

where

$A_\psi = 3$ for triaxial conditions

$$I_R = \frac{D_R}{100}\left[Q - \ln\left(\frac{100\sigma'_{mp}}{p_A}\right)\right] - R$$

$$\sigma'_{mp} = \frac{1}{3}\left(\sigma'_{1p} + 2\sigma'_{3p}\right)$$

Q=10

R=1